Superposition of Simple Harmonic Waves

This text provides background for the Java Applet on simple harmonic waves. A simple-harmonic wave can be written:

\[ A(t) = A_0 \sin(\omega t + \theta) \]  (1)

where \( A_0 \) is the maximum amplitude of the wave, and the angular frequency is \( \omega = 2\pi f \). This simple harmonic wave has a period of \( T = 1/f \). Consider two traveling, simple-harmonic waves:

\[ A_1(t) = A_{01} \sin(\omega_1 t + \theta_1) \]
\[ A_2(t) = A_{02} \sin(\omega_2 t + \theta_2) \]  (2)

Superimposing these two waves gives:

\[
A(t) = A_1(t) + A_2(t) \\
= (A_{01} + A_{02}) \sin(\bar{\omega} t + \bar{\theta}) \cos(\delta \omega t + \delta \theta) + (A_{01} - A_{02}) \sin(\delta \omega t + \delta \theta) \cos(\bar{\omega} t + \bar{\theta}) 
\]  (3)

where

\[
\bar{\omega} = \frac{\omega_1 + \omega_2}{2}, \quad \delta \omega = \frac{\omega_1 - \omega_2}{2} \\
\bar{\theta} = \frac{\theta_1 + \theta_2}{2}, \quad \delta \theta = \frac{\theta_1 - \theta_2}{2}. 
\]  (4)

If the two waves which have been added together have the same maximum amplitudes, \( A_{01} = A_{02} = A_0 \):

\[
A(t) = 2A_0 \cos(\delta \omega t + \delta \theta) \sin(\bar{\omega} t + \bar{\theta})
\]  (5)

The frequency, or pitch, of the combined wave is the mean frequency of the original two waves. The resulting wave has twice the maximum amplitude of the original waves, but this amplitude is modulated,

Modulated Amplitude = \( 2A_0 \cos(\delta \omega t + \delta \theta) \)  (6)