College Physics II

Electric Field & Electric Charge II

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January 2012
Overview

Electric Charge
Electrostatic Forces
Electric Fields
Review: Electric charges & forces

- Electric charge, conserved and quantized.
- Coulomb’s Law

Coulomb’s Law

- Attraction & Repulsion
- Adding Force Vectors

The Electric Field

- Electric Field Lines
- Electric Monopoles
- Electric Dipoles
With the exception gravity, almost every force that you witness in everyday life is electro-magnetic in origin.

- EM forces binding atoms together.
- EM forces binding atoms into molecules.
- EM forces bind molecules together.

Electric forces are produced by electric charges.
Electric Charge
Electric charge is quantized: A charged object has a surplus or deficit in the number of electrons relative to protons.

- $e =$ fundamental unit of charge

$$Q_{\text{proton}} = +e, \quad Q_{\text{electron}} = -e$$

- For any charge:

$$Q = ne, \quad n = 0, \pm1, \pm2, \ldots$$

SI Units: [Charge] = Coulomb = C

$$e = 1.6 \times 10^{-19} \text{ C.}$$
Electric charge is quantized: A charged object has a surplus or deficit in the number of electrons relative to protons. For any charged object:

\[ Q = n e, \quad n = 0, \pm 1, \pm 2, \ldots \]

- Let \( N_e \) be the number of electrons in an object
- Let \( N_p \) be the number of protons in an object.

\[ n = N_p - N_e \]
The total charge of any composite object is the sum of the charges of its charged constituents: electrons and protons.

- $N_e$ electrons, each with charge $-e$ contribute:
  \[ Q_{\text{electrons}} = N_e(-e) = -N_e e \]

- $N_p$ protons, each with charge $+e$ contribute:
  \[ Q_{\text{protons}} = N_p e \]

Together they give a total charge:
\[ Q = (N_p - N_e)e \]
Electric Charged is Conserved:

The total charge of an isolated system never changes.

For an Isolated System:

\[ Q_{\text{initial}} = Q_{\text{final}} \]
Examples of electric charge conservation:

**Pair production**

\[ \gamma + \gamma \rightarrow e^+ + e^- \]

**Pair annihilation**

\[ e^+ + e^- \rightarrow \gamma + \gamma \]

**Ionization**

\[ H + \gamma \rightarrow p^+ + e^- \]
Electrostatic Forces

The Electric Force

Active Learning

Adding Two Forces:

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

Example: Adding two forces
The Electric Force

Charges of opposite sign attract.

\[ F = k \frac{Q_1 Q_2}{r^2} = \frac{1}{4\pi \epsilon_0} \frac{Q_1 Q_2}{r^2} \]

Coulomb’s law:

\[ k \simeq 9 \times 10^9 \text{Nm}^2/\text{C}^2 \]

\[ \epsilon_0 = \text{permittivity of free space} \]

\[ \epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2 \]

Charges of the same sign repel.
Two equal mass pith balls are both charged. They hang on strings as shown in the figure. What can we say about the two charges charges?

- a. $Q_1 = Q_2$
- b. $Q_1 = -Q_2$
- c. $|Q_1| = |Q_2|$
- d. not enough information
Active Learning

Two equal mass pith balls are both charged. They hang on strings as shown in the figure. What can we say about the two charges charges?

- a. \( Q_1 = Q_2 \)
- b. \( Q_1 = -Q_2 \)
- c. \( |Q_1| = |Q_2| \)
- d. not enough information
Two equal mass pith balls are both charged. They hang on strings as shown in the figure. What can we say about the **sign** of the two charges?

a. One is + the other is −.

b. Both are −.

c. Both are +.

d. The charges are the same sign, but we can’t tell if they are + or −.
Two equal mass pith balls are both charged. They hang on strings as shown in the figure. What can we say about the sign of the two charges?

a. One is + the other is −.

b. Both are −.

c. Both are +.

d. The charges are the same sign, but we can’t tell if they are + or −.
Adding Two Forces: \( \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \)

**Algebraic Method**

- \( F_{1x} = F_1 \cos \theta_1 \)
- \( F_{1y} = F_1 \sin \theta_1 \)
- \( F_{2x} = F_2 \cos \theta_2 \)
- \( F_{2y} = F_2 \sin \theta_2 \)

- \( F_x = F_{1x} + F_{2x} \)
- \( F_y = F_{1y} + F_{2y} \)
- \( F = \sqrt{F_x^2 + F_y^2} \)
- \( \tan \theta = \frac{F_y}{F_x} \)

**Graphical Method**
Example: Adding two forces

Find the net force on \( q \):

\[
q = 3 \text{ C}
\]

\( Q_1 = 2 \text{ C} \)

\( Q_2 = -4 \text{ C} \)

\[
F = F_1 + F_2
\]

Algebraic Method

\[
F = k \frac{Qq}{r^2}
\]
Example: Adding two forces

Find the net force on \( q \):

\[ q = 3 \text{ C} \]

\[ Q_1 = 2 \text{ C} \]

\[ Q_2 = -4 \text{ C} \]

\[ F = F_1 + F_2 \]

Algebraic Method

\[ F_1 = k \frac{Q_1 q}{r^2} \]

\[ = (9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}) \frac{(2 \text{ C})(3 \text{ C})}{9 \text{ m}^2} \]

\[ = 6 \times 10^9 \text{ N} \]

\[ (F_1)_x = 6 \times 10^9 \text{ N}, \quad (F_1)_y = 0 \]
Example: Adding two forces

Find the net force on $q$:

![Diagram showing two charges, $Q_1 = 2 \, \text{C}$ and $Q_2 = -4 \, \text{C}$, separated by 3m. The net force $F = F_1 + F_2$.]

Algebraic Method

\[
(F_1)_x = 6 \times 10^9 \, \text{N}, \quad (F_1)_y = 0
\]

\[
F_2 = k \frac{|Q_2|q}{r^2} = \left(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right) \frac{(4 \, \text{C})(3 \, \text{C})}{18 \, \text{m}^2} = 6 \times 10^9 \, \text{N}
\]
Example: Adding two forces

Find the net force on \( q \):

\[ F = F_1 + F_2 \]

Algebraic Method

\[ (F_1)_x = 6 \times 10^9 \text{ N}, \quad (F_1)_y = 0 \]

\[ F_2 = 6 \times 10^9 \text{ N} \]

\[ (F_2)_x = -F_2 \cos 45^\circ = -4.24 \times 10^9 \text{ N} \]

\[ (F_2)_y = -F_2 \sin 45^\circ = -4.24 \times 10^9 \text{ N} \]
Example: Adding two forces

Find the net force on $q$:

\[ F = F_1 + F_2 \]

Algebraic Method

\[
(F_1)_x = 6 \times 10^9 \text{ N}, \quad (F_1)_y = 0
\]

\[
(F_2)_x = -F_2 \cos 45^\circ = -4.24 \times 10^9 \text{ N}
\]

\[
(F_2)_y = -F_2 \sin 45^\circ = -4.24 \times 10^9 \text{ N}
\]

\[
F_x = (F_1)_x + (F_2)_x = 1.76 \times 10^9 \text{ N}
\]

\[
F_y = (F_1)_y + (F_2)_y = -4.24 \times 10^9 \text{ N}
\]
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Electro-Magnetic Forces

Electric Charge

Electrostatic Forces

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Electric Field From a Point Charge
Monopole +
Monopole −
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Net Electric Field
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Electric Monopoles and Dipoles
The Capacitor
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Electric Fields
The electric field is the force/unit charge. Newtons/ Coulomb

\[ E = \frac{F}{q} \]

Electric field produced by \( Q \)

\[ E = k \frac{Q}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]

Compute the electric field, then find the force on \( q \):

\[ F = qE \]
Imaginary sphere of radius $r$

At a distance $r$ from $Q$:

$$E = \frac{1}{\epsilon_0} \frac{Q}{4\pi r^2}$$

- Surface Area: $4\pi r^2$
- Enclosed Charge $Q$
The electric field is a **vector** field. Magnitude:

$$E = k \frac{Q}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$$

where:

- $Q$ charge producing $\mathbf{E}$-field.
- $r$ distance between charge and point where $\mathbf{E}$ is measured.
- $\mathbf{E}$ points away from $+$ charge (source).
- $\mathbf{E}$ points toward $-$ charge (sink).
Electric Monopole Field +

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Positive charges are sources of electric field lines.

Negative charges are sinks of electric field lines.

Otherwise electric field lines are continuous.

The closer electric field lines are together, the stronger the electric field.

At any point, the electric field is tangent to the electric field lines.

Electric field lines do not cross.
Net Electric Field

\[ \mathbf{E} = k \frac{Q}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \quad \text{point charge} \]

For several charges add the electric field vectors produced by each charge.

\[ \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots \]

For example, consider two negative charges, \( Q_1 < 0 \) and \( Q_2 < 0 \), creating electric fields \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \) at a point. The net electric field \( \mathbf{E} \) is the vector sum of \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \).
A. Draw a picture showing the **electric field** at point $P$.

B. Draw a picture showing the **force** on a charge $q < 0$ if it is placed at point $P$.
A. Draw a picture showing the electric field at point P.

B. Draw a picture showing the force on a charge \( q < 0 \) if it is placed at point \( P \).
Active Learning

A. Draw a picture showing the **electric field** at point $P$.

B. Draw a picture showing the **force** on a charge $q < 0$ if it is placed at point $P$. 

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**Diagram:**

- A positive charge $Q$ is shown at point $P$.
- The electric field $E$ is shown as a vector from $Q$ to $P$.
- The force $F$ on the charge $q$ is shown as a vector from $Q$ to $P$.
Charges \(-Q\) and \(+Q\) are located on the \(x\)-axis as shown. What is \(E\) at point \(P\)?

A. \(k \frac{Q}{R^2} \left( \frac{1}{4} - 1 \right)\)

B. \(k \frac{Q}{R^2} \left( \frac{1}{25} - 1 \right)\)

C. \(k \frac{Q}{R^2} \left( \frac{1}{9} - \frac{1}{36} \right)\)

D. \(k \frac{Q}{R^2} \left( \frac{1}{25} + 1 \right)\)

E. None of the above
Charges $-Q$ and $+Q$ are located on the $x$-axis as shown. What is $E$ at point $P$?

A. $k \frac{Q}{R^2} \left( \frac{1}{4} - 1 \right)$

B. $k \frac{Q}{R^2} \left( \frac{1}{25} - 1 \right)$

C. $k \frac{Q}{R^2} \left( \frac{1}{9} - \frac{1}{36} \right)$

D. $k \frac{Q}{R^2} \left( \frac{1}{25} + 1 \right)$

E. None of the above
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Electric Field Lines

- **Positive** charges are sources of electric field lines.
- **Negative** charges are sinks of electric field lines.
- Otherwise electric field lines are **continuous**.
- The **closer** electric field lines are together, the **stronger** the electric field.
- At any point, the electric field is tangent to the electric field lines.
- Electric field lines do not cross.
Nature contains “+” and “-” electric charges.

- Electric dipole, every line from “+” ends on “-”.
- An Electric-dipole, has zero net charge.
The Capacitor

\[ E \]
The Capacitor

\[ F = qE \]

\[ \text{E} \]
The Capacitor

\[ F = qE \]

\( F \)
Next Time

Lecture 03: Electric Potential & Potential Energy

- Review: electric charges, forces and fields
- Electric potential (Voltage)
  - Electric potential is potential energy/unit charge
  - Potential difference & electric fields
  - Electric potential from point charges
  - Adding electric potentials
- Equipotential lines and $E$ field lines.
- The electron Volt (eV)