Quantization in Nature

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February, 2008
Overview

Electric Charge
Black Body Radiation
Photoelectric Effect
Compton Scattering
Outline

- Quantization of Charge
  - J.J. Thomson’s Experiment
  - Millikan’s Oil Drop
- Blackbody Radiation
  - Blackbody Radiation
  - Planck’s quantum hypothesis
- The photoelectric effect
- X-rays & Compton scattering
The Electron & Electric Charge
Quantization in Nature

◆ Many of the fundamental quantities in nature: matter, energy, electric charge come in indivisible “clumps”.

◆ These indivisible units are known as quanta. 1 quantum, 2 quanta, 3 quanta. . .

Quantization of Matter

◆ Democritus (450 B.C.)

◆ Avagadro’s hypothesis (1811).

Quantization of Charge

\[ q = ne, \quad e = 1.6 \times 10^{-19} \text{ C}, \quad n = 0, \pm 1, \pm 2, \ldots \]
Active Learning: Charge Quantization

Electric charge is quantized. A charged object has a surplus or deficit of the number of electrons relative to protons.

For any charged object:

\[ Q = ne, \quad n = 0, \pm 1, \pm 2, \ldots \]

- Let \( N_e \) be the number of electrons in an object.
- Let \( N_p \) be the number of protons in an object.
- Express \( n \) in terms of \( N_e \) and \( N_p \).
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\[ n = N_p - N_e \]
Charge & Charged Constituents

The total charge of any composite object is the sum of the charges of its charged constituents: electrons and protons.

- $N_e$ electrons, each with charge $-e$ contribute:
  
  $$Q_{\text{electrons}} = N_e(-e) = -N_e e$$

- $N_p$ protons, each with charge $+e$ contribute:
  
  $$Q_{\text{protons}} = N_pe$$

Together they give a total charge:

$$Q = (N_p - N_e)e$$

Example

$N_p = 3$
$N_e = 4$
$Q = -1e$
Quantization of Charge

Outline

Electric Charge
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Charge & Charged Constituents
Quantization of Charge
The Cathode Ray Tube (CRT)
Determination of $q/m$ of the electron
The Mass Spectrometer
The Millikan Oil Drop Experiment
Black Body Radiation

Photoelectric Effect
Compton Scattering

How do we know this?
The Cathode Ray Tube (CRT)

Cathode rays carry negative charge, J. Perrin 1895.
J.J. Thomson’s discovery of the electron in 1897.

\[ q/m = 1.76 \times 10^{11} \text{ C/kg} \]

Cathode ray tubes (CRT) are the vacuum tube display devices used in computer displays, video monitors, televisions and oscilloscopes, with the exception of LCD displays and plasma screens.
Determination of $q/m$ of the electron

\[ F = qvB = \frac{mv^2}{R} \]

\[ \frac{q}{m} = \frac{v}{BR} \]

Add a crossed electric field:

\[ F = q(E + v \times B) \]

adjust $E$ so that $F_{\text{net}} = 0$

\[ \frac{q}{m} = \frac{E}{B^2R} \]
The Mass Spectrometer

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\[ x = 2r = \frac{2}{B} \sqrt{\frac{2mV}{q}} \]
The Millikan Oil Drop Experiment

Robert Millikan’s oil drop experiment 1910-1913. For suspended drops:

\[ qE = Mg \]

\[ q = \frac{Mg}{E} \]

Studying motion at terminal velocity Millikan found:

\[ q = ne, \quad n = 0, \pm 1, \pm 2, \ldots \]

\[ e = 1.6 \times 10^{-19} \text{ C} \]
Equation of motion

\[ qE - Mg - bv = M \frac{dv}{dt}, \]

with \( b = 6\pi \eta r \), \( r \) = drop radius, \( \eta \) = coef. of viscosity.

Terminal velocity when \( a = \frac{dv}{dt} = 0 \)

\[ v = \frac{qE - Mg}{b} \]

Drops achieve terminal velocity immediately. Allow drops to drift upwards or downwards a distance \( L \). Rise time: \( v_r T_r = L \), fall time \( v_f T_f = L \)
Black Body Radiation
Blackbody radiation: thermal radiation emitted by opaque bodies. (perfect absorbers)

Oscillating, charged constituents of atoms and molecules produce EM radiation.

Josef Stefan 1879,
Stefan-Boltzmann Law - Radiated Power:

\[ R = \frac{P}{A} = \sigma T^4 \]

Stefan-Boltzmann constant

\[ \sigma = 5.7 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \]
Wein’s Displacement Law

\[ u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1} \]

Emission is a maximum at \( \lambda_m \) (Wilhelm Wien 1893)

\[ \lambda_m T = \text{constant} = 2.898 \times 10^{-3} \text{ mK} \]
The peak sensitivity of the human eye occurs near \( \lambda = 500 \text{ nm} \). Assuming this corresponds to \( \lambda_m \), compute the surface temperature of the sun.
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$$T_\odot = \frac{2.898 \times 10^{-3} \text{ m K}}{\lambda_m} = \frac{2.898 \times 10^{-3} \text{ m K}}{5 \times 10^{-7} \text{ m}}$$

$$T_\odot \approx 5,800 \text{ K}$$
The Rayleigh-Jeans Equation

Power radiated per unit area

\[ R = \frac{1}{4} c U \]

\( U \) = is the total energy density

Energy density spectral distribution function:

\[ u(\lambda) = kT n(\lambda) = 8\pi kT \lambda^{-4}, \]

\( n \) = the number of modes of oscillation per unit volume.
The Ultraviolet Catastrophe

Planck’s solution to the ultraviolet catastrophe:
Quantization of Energy

\[ E_n = n hf \]

quantum number \( n = 0, 1, 2, 3, \ldots \)

Rayleigh-Jeans:
\[ u(\lambda) = \frac{8\pi kT}{\lambda^4} \]

Planck’s law:
\[ u(\lambda) = \frac{8\pi hc}{\lambda^5} e^{\frac{hc}{\lambda kT}} - 1 \]

Planck’s constant

\[ h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \]
\[ = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s} \]
Planck’s Quantum Hypothesis

Simple Harmonic Oscillator

$$F = -kx$$

$$x(t) = A \cos \omega t$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Total energy (Classical)

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Planck’s solution:
Quantization of Energy

$$E_n = nhf$$

quantum number

$$n = 0, 1, 2, \ldots$$

Planck’s constant

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$
Discrete vs. Continuous Energies

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The Sun’s Surface Temperature
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\[ E = n hf \]

\( E \) varies continuously
Photoelectric Effect
Energy in EM Waves

Classically, the energy in an EM Wave is continuous, and independent of frequency. \( \bar{T} = \frac{1}{2} \varepsilon_0 c \mathbf{E}_0^2 \)
The Photo-electric effect

Light absorbed and electrons emitted.

\[ K_e = E_{\text{absorbed}} - \text{binding energy} \]
Your are a classical physicist from the late 19th century. What would you expect for the two plots below from 
\textbf{classical} physics?

\begin{align*}
\text{left plot: } & I = \text{const.} \\
\text{right plot: } & K_e = \text{const.}
\end{align*}
Your are a classical physicist from the late 19th century. What would you expect for the two plots below from classical physics?

\[ I = \text{const.} \quad \text{Ke} \]

\[ f = \text{const.} \]

\[ f \]

\[ I \]
Nature: Stranger than Fiction

Image how puzzled and confused these physicists were when experimental observation showed:

\[ I = \text{const.} \]

\[ f = \text{const.} \]
Electromagnetic radiation is **not** continuous.

- Light is quantized.
- These **quanta** are particles (photons).
- The energy of a photon is:

\[ E = hf \]

Planck’s Constant:

\[ h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s} \]
The Photo-electric effect

\[ K = eV_{\text{stop}} = hf - \Phi \]

\[ f_0 = \frac{\Phi}{h} \]

\[ V_{\text{stop}} = \frac{h}{e} f - f_0 \]
If the frequency is held fixed in the photoelectric effect and the intensity of the light is increases, which of the following are true:

a. The kinetic energy of the emitted electrons increases.

b. The kinetic energy of the emitted electrons stays the same.

c. The number of electrons emitted per second increases.

d. The number of electrons emitted per second stays the same.

e. (b) and (c)

f. (a) and (d)
Photoelectric Effect

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e. (b) and (c)

f. (a) and (d)
Compton Scattering
Electromagnetic radiation (light) is quantized. The indivisible particles which comprise light are called photons.

Energy of a photon:

\[ E = hf \]

Momentum of a photon:

\[ p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \]
Compton Scattering

1923 A.H. Compton scatters X-rays from various metals:

Classically

$$\Delta \lambda = \lambda' - \lambda = 0$$

Quantum mechanically:

$$p_\gamma = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Compton wavelength of the electron:

$$\lambda_c = \frac{h}{m_e c}$$

$$\Delta \lambda = \frac{h}{m_e c} \left(1 - \cos \phi\right)$$
Elastic Scattering: $\gamma + e^- \rightarrow \gamma + e^-$

**Before**

\[ \begin{array}{ccc}
\gamma & \rightarrow & e^- \\
\lambda & \rightarrow & \lambda
\end{array} \]

**After**

\[ \begin{array}{ccc}
e^- & \rightarrow & \lambda' \\
\phi & \rightarrow & \phi
\end{array} \]

**Electron energy & momentum:**

\[ E_e^2 = (p_e c)^2 + (m_e c^2)^2 \]

**Conservation of energy:**

\[ h\lambda f + m_e c^2 = h\lambda' f' + E_e \]

**Conservation of momentum:**

\[ \begin{align*}
(p_x) & \quad \frac{h}{\lambda} = \frac{h}{\lambda'} \cos \phi + p_e \cos \theta \\
(p_y) & \quad 0 = \frac{h}{\lambda'} \sin \phi - p_e \sin \theta
\end{align*} \]

**Photon energy & momentum:**

\[ E_\gamma = p_\gamma c = h\lambda = \frac{hc}{\lambda} \]
Sample Photon Interactions

- **Absorption & Ionization, *i.e.* photoelectric effect:**
  \[ \sim \rightarrow \quad \Rightarrow \quad \rightarrow \]

- **Absorption without ionization *e.g.*, \( \gamma + H \rightarrow H^* \)
  \[ \sim \rightarrow \quad \Rightarrow \quad \rightarrow \]

- **Scattering, *e.g.* Compton scattering
  \[ \sim \Rightarrow \quad \rightarrow \quad \rightarrow \]

- **Pair Production or annihilation**
  \[ e^+ + e^- \rightarrow \gamma + \gamma \]
Lecture 07: Quantization II

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- X-rays & Compton scattering