The Nuclear Atom

Rutherford Scattering

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Overview

Atomic Spectra
Alpha Scattering
The Bohr Model
Atomic Spectra

Continuous and Discrete Spectra
Balmer’s Formula (1885)
The Rydberg-Ritz Formula
Emission and Absorption Spectra
Atomic Spectra
Thomson’s Plum Pudding

Alpha Scattering

The Bohr Model
Continuous and Discrete Spectra

- Continuous Spectra
  
  ![Continuous Spectra Graph]

- Line Spectra
  
  ![Line Spectra Graph]
Balmer’s Formula (1885)

The Balmer Series (visible and ultraviolet):

Simple empirical formula for visible hydrogen lines:

$$\lambda_n = 3.64 \frac{n^2}{n^2 - 4} \text{ nm}$$

Reciprocal form:

$$\frac{1}{\lambda_n} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right), \quad R = 1.097 \times 10^7 \text{ m}^{-1}$$
The Rydberg-Ritz Formula

\[
\frac{1}{\lambda_{mn}} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right), \quad (m < n < \infty)
\]

\[R = 1.097 \times 10^7 \text{ m}^{-1}\] Rydberg constant

\[
E_n - E_m = 13.6 \text{ eV} \left( \frac{1}{m^2} - \frac{1}{n^2} \right)
\]

\[
E_{\gamma} = hf = \frac{hc}{\lambda} = E_n - E_m
\]
Emission and Absorption Spectra

Atomic Spectra
Continuous and Discrete Spectra
Balmer’s Formula (1885)
The Rydberg-Ritz Formula

Emission and Absorption Spectra
Atomic Spectra
Thomson’s Plum Pudding

Alpha Scattering
The Bohr Model

Gas

continuous spectrum

absorption line-spectrum

emission line-spectrum
Atomic Spectra

- Atomic Spectra (CTC)
- Spectra Applet (Colorado)
- Spectra Applet (Oregon)
Electrons embedded in a homogeneous, positively charged mass
Alpha Scattering

Scattering and the Cross section
Differential Cross Section
Alpha Scattering
Alpha Scattering (Plot)
Geiger & Marsden’s Experiment
Plot
Rutherford’s Astonishment
Elastic Collisions
Rutherford’s Atom
Geiger & Marsden’s Experiment
Alpha Scattering
Alpha Scattering (Plot)
Plot

The Bohr Model

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Scattering and the Cross section

- $\sigma$ - effective area of target
- $N_I$ - No. of incident particles
- $N_e$ - No. of scattering events
- $n$ - No. of targets/unit volume

Probability of hitting target with one particle:

$$P = \frac{\text{area of targets}}{\text{total area}} = \frac{\sigma(nV)}{A} = \frac{\sigma(nxA)}{A} = \sigma nx$$

Number of Scatterings:

$$N_e = N_I(nx)\sigma$$
**Differential Cross Section**

**Total Cross Section**

\[ \sigma = \pi b^2 \]

**Differential Cross Section**

\[ d\sigma = \left( \frac{d\sigma}{d\Omega} \right) d\Omega \]

**Solid Angle:** \[ [\Omega] = \text{sterradians} \]

\[ d\Omega = \sin \theta d\theta d\phi, \quad \Omega_{\text{sphere}} = 4\pi \]
Alpha Scattering

$\alpha$ kinetic energy:

$$E_k = \frac{1}{2} m_\alpha v^2$$

Impact parameter:

$$b = \frac{k q_\alpha Q}{2 E_k} \cot \frac{\theta}{2}$$

Cross Section:

$$\sigma = \pi b^2$$

Differential cross section:

$$\frac{d\sigma}{d\Omega} = \left( \frac{kqQ}{4E_k} \right)^2 \frac{1}{\sin^4(\theta/2)}$$
\[
\frac{d\sigma}{d\Omega} = \left(\frac{kgQ}{4E_k}\right)^2 \frac{1}{\sin^4(\theta/2)}
\]
Radioactive source $^{214}$Bi. Screen coated with scintillator (zinc sulfide). Scintillations viewed through microscope. Experiment counted the number of scintillations vs. $\theta$. 

Pb shield

Au foil

Scintillation Screen

$\alpha$ beam

$\theta$
Geiger and Marsden’s Results

\[ \ln(N) \]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( N )</th>
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<tbody>
<tr>
<td>15.0</td>
<td>132,000</td>
</tr>
<tr>
<td>22.5</td>
<td>27,300</td>
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<tr>
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<td>45.0</td>
<td>1,457</td>
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<td>60.0</td>
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<tr>
<td>75.0</td>
<td>211</td>
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<td>105.0</td>
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<td>135.0</td>
<td>43</td>
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scattering angle \( \theta \)
Experimentally, a very small fraction of the $\alpha$ particles bounced back.

Later, Rutherford expressed his astonishment:

“It was almost as incredible as if you fired a fifteen inch (artillery) shell at a piece of tissue paper and it came back to hit you.”
Elastic Collisions
Rutherford’s Atom

Atom: $10^{-10}$ m

Nucleus: $10^{-15}$ m.
Radioactive source $^{214}\text{Bi}$. Screen coated with scintillator (zinc sulfide). Scintillations viewed through microscope. Experiment counted the number of scintillations vs. $\theta$
Alpha Scattering

$\alpha$ kinetic energy:

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$$b = \frac{k q_\alpha Q}{2 E_k} \cot \frac{\theta}{2}$$

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Differential cross section:

$$\frac{d\sigma}{d\Omega} = \left( \frac{k q Q}{4 E_k} \right)^2 \frac{1}{\sin^4(\theta/2)}$$
Alpha Scattering (Plot)

\[
\frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4E_k}\right)^2 \frac{1}{\sin^4(\theta/2)}
\]

scattering angle \(\theta\)
Geiger and Marsden’s Results

\[ \ln(N) \]

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scattering angle \( \theta \)
The Bohr Model

Atomic Spectra

Alpha Scattering

The Bohr Model

Planetary Model

Bohr’s Model of the Atom (1915)

Bohr’s Model of the Atom (II)

Bohr Quantization Condition

$L = n\hbar$

Fine Structure Constant

Hydrogen

Energy Levels

Absorption of Photons

Emission of Photons

Hydrogen Transitions

Fluorescence

The Correspondence Principle
Planetary Model

Bohr’s Model of the Atom (1915)
- Bohr’s Model of the Atom (II)
- Bohr Quantization Condition

\[ L = n \hbar \]

Fine Structure Constant

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Next
Classical Planetary Model

Total Energy of the system:

\[ E = K + U = \frac{1}{2}mv^2 - k\frac{Ze^2}{r} \]

Newton’s Second Law for \textit{circular} orbits:

\[ F = k\frac{e^2Z}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = k\frac{Ze^2}{mr} \]

Virial Theorem

\[ K = -\frac{1}{2}U, \quad \Rightarrow \quad E = K + U = \frac{1}{2}U = -K = -mv^2 \]

For later use:

\[ v^2 = \frac{kZe^2}{2mr}, \quad E = -k\frac{Ze^2}{2r} \]
Bohr’s Model of the Atom (1915)

- Angular momentum is quantized.

\[ L = n \frac{\hbar}{2\pi} = n\hbar \]

\[ L = mvr = n\hbar \quad \Rightarrow \quad v = \frac{n\hbar}{mr} \]

- Emission and absorption of electromagnetic radiation only occurs in a transition between states

\[ E = E_i - E_j = hf \]
Bohr’s Model of the Atom (II)

\[ E = K + V = \frac{1}{2}mv^2 - k\frac{e^2 Z}{r} = -k\frac{Ze^2}{2r} \]

\[ L = mvr = n\hbar \quad \Rightarrow \quad v^2 = \left(\frac{n\hbar}{mr}\right)^2 = \frac{kZe^2}{2mr} \]

**Allowed Radii:**

\[ r_n = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2}{Z}a_0 \]

**Bohr radius**

\[ a_0 = \frac{\hbar^2}{mk\epsilon^2} = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.53 \times 10^{-10} \text{ m} \]

**Energy:**

\[ E_n = -k\frac{Ze^2}{r_n} = -\frac{m(kZe^2)^2}{2\hbar^2n^2} \]
Bohr Quantization Condition $L = n\hbar$

Atomic Spectra
- $n = 1$
  - $1\hbar = pr$

Alpha Scattering
- $n = 2$
  - $2\hbar = pr$

The Bohr Model
- Planetary Model
- Bohr's Model of the Atom (1915)
- Bohr's Model of the Atom (II)

Bohr Quantization Condition $L = n\hbar$
- Fine Structure Constant
- Hydrogen
- Energy Levels
- Absorption of Photons
- Emission of Photons
- Hydrogen Transitions
- Fluorescence
- The Correspondence Principle

Circular standing Waves
- $n = 3$
  - $3\hbar = pr$
- $n = 4$
  - $4\hbar = pr$
Fine Structure Constant

Fine structure constant $\alpha = \nu_1/c$:

$$\alpha = \frac{ke^2}{\hbar c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

Bound state energies

$$E_n = -\frac{1}{2}m_e c^2 \frac{Z^2\alpha^2}{n^2} = -\frac{Z^2(13.6 \text{ eV})}{n^2}$$

Higher Order Corrections

$$E_{nj} = E_n \left[ 1 + \left( \frac{Z\alpha}{n} \right)^2 \left( \frac{n}{j + \frac{1}{2}} - \frac{3}{4} \right) \right]$$
Quantized energy levels

\[ E_n = -\frac{2\pi^2 Z^2 e^4 m k^2}{\hbar^2 n^2} \]

\[ = -E_0 \frac{Z^2}{n^2} \]

\[ = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \]

\( n = \) principle quantum number.

◆ Hydrogen Applet
Absorption of Photons

Hydrogen energy levels

\[ E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \]

Conservation of Energy:

\[ E_{n_i} + hf = E_{n_f} \]

Photon frequency

\[ f = \frac{Z^2 E_0}{h} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \]
Absorption of Photons

Hydrogen energy levels

\[ E_n = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \]

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Emission of Photons

Atomic energy levels

\[ E_n = -E_0 \frac{Z^2}{n^2} = -(13.6 \text{ eV}) \frac{Z^2}{n^2} \]

Photon emitted: \( n_i \rightarrow n_f < n_i \)

\[ E_{n_i} = E_{n_f} + hf \]

\[ f = \frac{Z^2 E_0}{\hbar} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]
Emission of Photons

Atomic energy levels

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Hydrogen Transitions

Atomic Spectra
Alpha Scattering

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Hydrogen Energy Levels
Absorption of Photons
Emission of Photons

Hydrogen Transitions
Fluorescence
The Correspondence Principle
Next

Energy (eV)

Lyman Series
Paschen Series
Balmer Series

$n = \infty$
$n = 4$
$n = 3$
$n = 2$
$n = 1$
Fluorescence

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In the limit of large orbits and large energies, quantum calculations must agree with classical calculations.
Lecture 09: The Nuclear Atom II

- The Nuclear Atom
  - Atomic Spectra
  - Rutherford’s Nuclear Model
  - The Bohr Model
  - X-Ray Spectra
  - The Franck-Hertz Experiment