Pre-Lab 7 Assignment:

Capacitors and RC Circuits

(Due at the beginning of lab)

Directions: Read over the Lab Handout and then answer the following questions about the procedures.

**Question 1** How do you think the capacitance of a parallel plate capacitor changes as the area of the plates is increased?

**Question 2** How do you think the capacitance of a parallel plate capacitor changes as the separation between the plates is increased?

**Question 3** What observations will you make in activity 1-2 to test your answer either to question 1 or 2.

**Question 4** What do you predict is the net capacitance of two identical capacitors connected in parallel?

**Question 5** What observations will you make in activity 2-1 to test this prediction?

**Question 6** Sketch the complete circuits in Fig. 5 (a) with the switch in position 1, and (b) with the switch in position 2.
LAB 7 // Capacitors and RC Circuits

Objectives

• To define capacitance and learn how to measure it.

• To discover how the capacitance of a parallel plate capacitor is related to the area of the plates and the distance separating them.

• To examine how the net capacitance in a circuit is related to the capacitances of individual capacitors when several capacitors are wired in parallel or in series through the process of conceptual reasoning as well as through direct measurements.

• To explore the effect of connecting a capacitor in a circuit in series with a resistor or light bulb and a battery.

• To discover how the electric charge on a capacitor and the electric current in a circuit change with time when a charged capacitor is placed in a circuit in series with a resistor.

This lab has been adapted from the Real Time Physics Active Learning Laboratories [?]. The goals, guiding principles and procedures of this lab closely parallel the implementation found in the work of those authors [? , ? , ?].

Overview

Two conducting surfaces that are separated by an insulator can be electrically charged in such a way that one conductor has a positive charge and the other conductor has an equal amount of negative charge; such an arrangement is called a capacitor. A capacitor can be made up of conductors of regular or irregular shapes such as a hollow cylinder inside another, or two blobs of metal separated by a distance.

![Cylindrical Capacitor](image1.png)

![Parallel Plate Capacitor](image2.png)

Figure 1: Some different capacitor geometries.

The type of capacitor most commonly used as prototype is the parallel plate capacitor. The capacitance of a parallel plate capacitor is related to the geometry of the capacitor through a simple relationship and is a relatively easy device to construct. Capacitors are widely used
in electronic circuits that require storage of electrical energy (camera flash) or a specific time-dependent feature (e.g., intermittent wipers). Finally, although capacitors show some very interesting properties when connected in alternating current circuits, for this lab, we will limit ourselves to the behavior of capacitors in direct current circuits like those you are familiar with from the lab exercises of the last few weeks. The circuit symbol for a capacitor is a pair of lines as shown in Fig. 2.

![Figure 2: The circuit diagram symbol for a capacitor.](image)

**Investigation 1:**

**Capacitance, Area, and Separation**

The easiest method for transferring equal and opposite charges onto the plates of a capacitor (charging the capacitor) is to use a voltage source such as a battery or a power supply to create a potential difference between the two conducting plates. This potential difference causes electrons to flow from one conductor (leaving positive charges behind) and onto the other until the potential difference between the two conductors is the same as that provided by the voltage source. In general, the amount of charge transferred before this condition is met depends on the size, shape, and location of the conductors relative to each other. The capacitance of the given capacitor is defined mathematically as the ratio of the magnitude of the charge, $q$, on either one of the conductors to the voltage, $V$, applied across the two conductors so that:

$$C = \frac{q}{V}$$

In words, capacitance is a measure of the amount of excess charge on either one of the conductors per unit potential difference between the plates.

Draw on your understanding of potential difference to examine what might happen to the charge as it flows in a circuit containing a parallel plate capacitor and a battery as shown in Fig. 3. Such a thought experiment can help you develop an intuitive understanding for the meaning of capacitance. For a fixed voltage from a battery, the net charge found on either plate is proportional to the capacitance of the capacitor.

**Activity 1.1: Predicting the dependence of Capacitance on Area and Separation**

To visualize our circuit with a parallel plate capacitor, consider two identical metal plates of area $A$, separated by a non-conducting material that has a thickness $d$. They are connected to a battery through a switch, as shown above. When the switch is open, there is no excess charge on either plate. The switch is then closed completing the path between the battery and the capacitor.

**Question 1.1** What does the reasoning you pursued in your thought experiment predict will happen to the conducting plate attached to the negative terminal of the battery? What do you
Figure 3: A parallel plate capacitor in series with a battery and switch: (a) pictorial representation, (b) circuit diagram.

predict will happen to the conducting plate that is connected to the positive terminal of the battery? Explain.

Question 1.2 Can excess charges on one plate of a charged parallel plate capacitor exert forces on the excess charges on the other plate? If so, how?

Question 1.3 What limits the amount of charge that will flow onto either plate?

Question 1.4 Use qualitative reasoning to predict how the amount of charge that a pair of parallel plate conductors can hold changes as the area of the plates is increased. What does this imply about the capacitance?

Question 1.5 What happens to the electric field between the plates when the spacing, d, between the plates is increased? Will the magnitude of the electric field increase, decrease or stay the same? Explain.
**Question 1.6** Does the amount of charge a given battery can pile onto the plates of a capacitor increase or decrease as the spacing, \(d\), between the plates of the capacitor increases? What does this imply about the capacitance?

The unit of capacitance is the farad, F, named after Michael Faraday. One farad is equal to one coulomb/volt. One farad is a very large capacitance and actual capacitances are usually expressed in smaller units with the notation shown below:

- **microfarad**: \(10^{-6}\) F = 1 \(\mu\)F
- **nanofarad**: \(10^{-9}\) F = 1 nF = 10\(^{-3}\) \(\mu\)F = 10\(^3\) \(\mu\)\(\mu\)F
- **picofarad**: \(10^{-12}\) F = 1 pF = 10\(^{-6}\) \(\mu\)F = 1 \(\mu\)\(\mu\)F

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion</th>
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<tr>
<td>microfarad</td>
<td>(10^{-6}) F = 1 (\mu)F</td>
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<td>nanofarad</td>
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<td>picofarad</td>
<td>(10^{-12}) F = 1 pF = 10(^{-6}) (\mu)F = 1 (\mu)(\mu)F</td>
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Table 1: Units of capacitance

Technical note: Because of the occasional lack of availability of Greek fonts to capacitor manufacturers, sometimes the symbol \(m\) is used as a label on capacitors instead of the symbol \(\mu\) to represent \(10^{-6}\), despite the fact that in other situations \(m\) always represents \(10^{-3}\).

In the next few activities you will construct a parallel plate capacitor and use a multimeter to measure its capacitance. You will need the following items:

- 2 pieces aluminum foil, approximately 20 cm \(\times\) 20 cm
- 1 thick textbook
- 1 digital multimeter (with a capacitance mode)
- 2 insulated wires (stripped at the ends, approximately 12” long)
- 1 ruler

Use two rectangular sheets of aluminum foil separated by pieces of paper to make your parallel plate capacitor. Leaving small tabs in the aluminum sheets that can stick out provides an easy way to connect the leads from the multimeter to the sheets of aluminum foil. Use the textbook pages as the separator for the foil; the textbook pages constitute a convenient insulator. Use not more than 8 pages between the two foil sheets. Press down on the book to make sure the pieces of aluminum foil are smoothed out. Then use your digital multimeter in its capacitance mode for the measurements before connecting the capacitor to a power source. Warning: When you use the multimeter to measure the capacitance of your lab-made parallel plate capacitor, make sure that the capacitor is not connected to any source of power. This is very important, otherwise the multimeter can get damaged.

**Activity 1.2: Measuring how the capacitance depends on area of the plates or how capacitance depends on the distance of separation between the plates**

**Step 1:** Decide whether you are going to determine how the capacitance depends on the foil area or alternately, how the capacitance depends on the separation between the foil sheets.
**Step 2:** If you and your lab partner decided that you will examine the relationship between the capacitance and the distance of separation between the aluminum foil sheets, describe how you will change the separation and keep the area constant. On the other hand, if you decided to examine the relationship between the capacitance and the area of the foil sheets, describe how you will vary the area while keeping the separation constant. In both cases, describe in detail all of the quantities that you will need to measure for each data point. Take at least five data points in either case and record your data in Table 2. Caution: When measuring the capacitance of your "parallel plate capacitor" make sure that the sheets of aluminum foil don’t touch each other and "short out."

<table>
<thead>
<tr>
<th>Separation (m)</th>
<th>Length (m)</th>
<th>Width (m)</th>
<th>Area (m²)</th>
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Table 2:

**Step 3:** After collecting all of the required data, open the relevant file for the activity you chose to do i.e. either experiment file L7A1-2a (Capacitance vs Separation) or experiment file L7A1-2b (Capacitance vs Area). Enter your data for the capacitance and either the separation distance or the foil area, depending on which investigation you chose, from table 2. Write in the appropriate labels on the vertical and horizontal axes shown below, and give the vertical and horizontal scales appropriate values. Draw your graph using these axes.

**Step 4:** If your graph looks like a straight line, use the linear fit command in the analysis menu. If not, try other functional relationships until you find the best fit for your data points.
Question 1.7 What function best describes the relationship between capacitance and spacing or capacitance and foil area? How do the results you obtain from an analysis of the graph compare with those that you predicted based on conceptual reasoning (your answers to questions 1.4 and 1.6)?

The quantitative relationship between the area of the plates $A$, the separation of the plates $d$, and the capacitance, $C$, of a parallel plate capacitor is

$$C = \kappa \epsilon_0 \frac{A}{d}$$

where $A$ is the area of the plates $d$ is their separation, and $\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2$ is the permittivity of free space.

The quantity $\kappa$ (kappa) is called the dielectric constant and is a property of the insulating material that separates the two plates. $\kappa = 1$ for air and is greater than 1 for other materials. Paper, the insulating material we have used, has a dielectric constant of between 1.5-4.0

Question 1.8 Do the predictions you made as well as your observations agree qualitatively with this relation? What does this relation indicate will happen when the area of the plates increases? Or when the separation increases? Explain.

Question 1.9 Use one of your actual areas and distance of separation from the measurements you made in Activity 1-2 to calculate a value of $C$. Assume that the dielectric constant, $\kappa$ for paper is about 3.5. How does the calculated value of $C$ compare with the directly measured value?

Question 1.10 Let’s apply this relation to see how realistic a capacitance of 1 F is. You want to make a parallel place capacitor like the one you just made, but with a capacitance of 1 F. If you were to use two square foil sheets, separated by paper with a dielectric constant of 3.5 that is 1 mm thick, how long (in miles) would each side of the square foil sheet have to be in order to have capacitance $C = 1$ F? Show all your calculations. Note: 1000 m = 1 kilometer = 0.62 miles.

$$L = \text{__________} \text{miles}$$

Investigation 2:
Capacitors in Series and Parallel

Two capacitors can be combined either in series or in parallel in a circuit, as shown in Fig. 4. The terms series and parallel refer to the same arrangements that you carried out for other
circuit elements such as resistors. In a series combination there is only one path for the flow of electric charge, and whatever charge is placed on one capacitor must also be transferred to the other. In a parallel connection, the two terminals of each capacitor are connected to the terminals of the other. While each capacitor has the same potential difference across it, each constitutes a branch so that the total charge transferred to the capacitor combination is split among the two capacitors.

To examine the equivalent capacitance of two capacitors connected in series or in parallel you will need:

- 2 capacitors, on the order of 0.1 \( \mu \text{F} \)
- 1 multimeter with capacitance range.
- Pasco circuit board
- connecting wires.

Pasco Circuit Board:
Activity 2.1: Equivalent Capacitance of Capacitors in Parallel

Prediction 2.1  Reason using your understanding of capacitance to predict the equivalent capacitance of a pair of identical capacitors wired in parallel. Explain your reasoning. Hint: What is the effective area of two parallel plate capacitors wired in parallel in Fig. 4? Does the effective spacing between plates change?
Step 1: Measure the capacitance of each of your capacitors with the multimeter.

\[ C_1 = \quad \text{F} \quad \quad C_2 = \quad \text{F} \]

Step 2: Connect the two capacitors in parallel, and measure the equivalent capacitance of the parallel combination using a multimeter.

\[ C_{\text{eq}} = \quad \text{F} \]

Question 2.1 Using your measured data, propose a general equation for the equivalent capacitance \( C_{\text{eq}} \) of two capacitors connected in parallel in terms of the individual capacitances \( C_1 \) and \( C_2 \). Be sure that your measured values fit your equation.

\[ C_{\text{eq}} = \]

Question 2.2 Does the form of this equation agree with your prediction 2.1?

Activity 2.2: Equivalent Capacitance of Capacitors in Series

Prediction 2.2 Reason using your understanding of capacitance to predict the equivalent capacitance of two capacitors wired in series. Explain your reasoning. Hint: If you connect two capacitors in series, what will happen to the charges along the conductor between them? What will the effective separation of the "plates" be? Will the effective area change?

Step 1: Measure, using a multimeter, the equivalent capacitance when two lab-made capacitors (two pairs of aluminum sheets) or two store-bought capacitors are wired in series.

\[ C_{\text{eq}} = \quad \text{F} \]

Question 2.3 Are your measurements consistent with the following equation for the equivalent capacitance of two capacitors connected in series? Show the calculations you used to reach your conclusion.

\[ \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \]

Question 2.4 Does the equation given in Equation 2.3 agree with your Prediction 2.2?
**Question 2.5** How do the quantitative relationships for series and parallel capacitors compare to those of resistors? Do series capacitors combine more like series resistors or parallel resistors?

**Investigation 3:**

**Charge Build-up and Decay in Capacitors**

Capacitors can be connected with other circuit elements such as resistors. Connecting capacitors in series with resistors in a circuit gives rise to interesting time-dependent phenomena. In this last activity you will investigate what happens to the voltage across a capacitor when it is placed in series with a resistor in a direct current circuit. Use your observations to come up with qualitative as well as quantitative explanations of what happens in these circuits.

For the activities in this investigation you will need:

- computer-based laboratory system.
- experiment configuration files.
- differential voltage probe.
- current probe.
- 2 D-cell 1.5 volt batteries.
- Pasco circuit boards
- flashlight bulb
- 33000 \(\mu\)F (or larger) capacitor.
- 3300 \(\mu\)F (or larger) capacitor.
- 10 \(\Omega\) resistor
- 100 \(\Omega\) resistor.

**Activity 3.1: Observations with a Capacitor, Battery and Bulb**

**Step 1:** Set up the circuit shown in Fig. 5 with a capacitor with a capacitance of at least 33,000 \(\mu\)F. (If you are using a polar capacitor, make sure that the positive and negative terminals of the capacitor are connected correctly.)

**Question 3.1** Sketch one-loop circuits equivalent to the circuit in Fig. 5, showing just the portion of the circuit that current flows in when the switch is in position 1, and the portion of the circuit that current flows in when the switch is in position 2.
Step 2: Move the switch to position 2. After at least 10 seconds, switch it to position 1. Describe, in detail, what happens to the brightness of the bulb.

**Question 3.2** Draw a qualitative sketch on the axes below of the approximate brightness of the bulb as a function of time where you move the switch to position 1 after it has been in position 2 for a long time. Let $t = 0$ be the time when the switch was moved to position 1.
Step 3: Next, move the switch back to position 2. Describe, in detail, what happens to the brightness of the bulb. Note that this removes the battery from the circuit, but completes a circuit that contains just the capacitor and the light bulb. Did the bulb light again without the battery in the circuit? If so, where did the bulb get the electrical energy from?

**Question 3.3** *Draw a qualitative sketch on the axes below of the approximate brightness of the bulb as a function of time when it is connected to the capacitor without the battery, i.e., when the switch has been moved to position 2. Start with \( t = 0 \) as the time when the switch was moved to position 2.*

[Sketch of a graph with axes labeled 'Brightness' and 'Time']

**Question 3.4** *Explain why the bulb behaves in this fashion. Is there a charge on the capacitor after the switch is in position 1 for a little while? What happens to this charge when the switch is moved back to position 2?*

Step 4: Open the experiment file called Capacitor Decay (L7A3-1), and display the axes shown on the next page.

Step 5: Connect the current probe and the voltage probe to the interface, and zero the probes with nothing attached to them.

Step 6: Connect the current and voltage probes as shown in Fig. 6 so that the current probe measures the current through the light bulb and the voltage probe measures the potential difference across the capacitor.

Step 7: Next, move the switch to position 2. Begin taking the data. When the graph lines appear on the screen, move the switch to position 1. When the current and voltage data stop changing with time, move the switch back to position 2.
Step 8: Sketch the graphs on the axes above. Indicate on the graphs the times at which the positions of the switch was changed, i.e. the time at which the switch was moved from position 2 to position 1, and the time at which it was moved back to position 2 again.

Question 3.5 Does the actual behavior over time observed on the current graph agree with your sketches in questions 3-2 and 3-3? Comment on any features of the graph that you did not expect or account for in your sketch. Explain.

Question 3.6 Use the graph of potential difference across the capacitor to explain why the bulb lights up when the switch is moved from position 1 to position 2 even though in this position of the switch the bulb is connected to the capacitor with no battery in the circuit. Also explain why the brightness of the bulb changes with time.

As you have seen in the lab activity on understanding resistance, a light bulb does not have a constant resistance. Instead, its resistance is temperature dependent and increases when the
current through the filament heats it up. For more quantitative studies of the behavior of a circuit with resistance and capacitance, you should replace the bulb with a 10 Ω resistor.

Activity 3.2: The Rise and Decay of a voltage in an RC Circuit

Step 1: Replace the light bulb in the circuit shown in Fig. 6 with a 10 Ω resistor. Move the switch to position 2. Begin taking the data. When the graph lines appear on the screen, move the switch to position 1. When the current and voltage data stop changing with time, move the switch back to position 2.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Current 1 (A)</th>
<th>Voltage 2 (V)</th>
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Step 2: Sketch the graphs on the axes above. Indicate on the graphs the times at which the positions of the switch was changed, i.e. the time at which the switch was moved from position 2 to position 1, and the time at which it was moved back to position 2 again.

Question 3.7 Do the graphs you sketched for the resistor appear similar to those for the light bulb? Are there any significant differences?

The time interval for the voltage across the capacitor to decay to 37% of its initial value is known as the time constant. The time constant of a circuit containing a resistor and a capacitor (commonly called a RC circuit) depends on the values of the capacitance and resistance in the circuit.

Step 3: Use the Examine feature in the Analysis menu to determine the time and the initial voltage across the capacitor at the moment the switch was moved to position 2. Then determine the time at which the voltage was 37% of this initial value. You have now determined the time constant of your RC circuit.

Initial voltage: ____________ time: ____________
37% of initial voltage: _________ time constant: _________

Careful measurements of the dependence of the potential difference V across a capacitor vs. time t, for a capacitor (C) discharging through a resistor (R) should reveal an exponential decay behavior for the potential difference across the capacitor. Such exponential
forms for decay or growth describe many physical and natural phenomena, such as radioactive decay, the concentration of a reactant in a chemical reaction and the growth of bacterial colonies.

Mathematical reasoning based on the application of the definitions of resistance, current and capacitance can be used to show that the following equation represents the potential difference across the capacitor, \( V \), as a function of time:

\[
V(t) = V_0 e^{-t/RC}
\]

In this equation, \( V_0 \) is the initial potential difference across the capacitor. (Note that \( V_0 \) is not necessarily the voltage of the battery.)

**Step 4:** Utilize the fit routine in the software to examine whether an exponential decay accurately represents the behavior of the voltage across the capacitor as a function of time.

Make sure you select and fit only that portion of the voltage graph where the voltage is decaying (decreasing) - up to the point where the voltage just reaches its minimum value.

**Question 3.8** Record the equation as determined by the fit routine you used. How well did the exponential function fit your data? Is the decay of voltage across the capacitor an exponential decay?

**Question 3.9** Find the value of \( RC \) from the exponent in the function that fit the data. Compare this value to the one you calculated from the values of resistance and capacitance. Do they agree? (Note that the resistance and capacitance values are each only known to an accuracy (tolerance) of about 10%.)

When \( t = RC \) in the equation above, \( V = V_0 e^{-1} = 0.37V_0 \). Thus \( RC \) is equal to the time constant.

**Question 3.10** How does the time constant you determined in step 3 compare with the value of the time constant you obtained by using the fit procedure in the later part of the above activity?

If you have time after the above activities, do the following extension.

**Activity 3.3: Extension: Decay with Other Resistances and Capacitances**

**Prediction 3.1** How will the discharging behavior of a capacitor change if the capacitance is made smaller? How do you think it will affect the time constant?
Test your prediction.

**Step 1:** Replace the 1F capacitor with the 3300 $\mu$F capacitor. Note: Since this is a polarized capacitor, make sure you connect the positive end of the capacitor as shown in Fig. 6.

**Step 2:** Start with the switch in position 2. Begin taking the data. When the graph lines appear on the screen, move the switch to position 1. Then move the switch to position 2.

**Step 3:** Since this capacitor is considerably smaller than the one you first used, the decay is significantly faster. Change the time scale on your graph so that you can clearly see the decay of the voltage and current.

**Step 4:** Select the portion of your graph that shows the decay after you moved the switch to position 2. Record the equation as determined by the fit routine you used.

**Question E3.1** Use the constant in the exponent of the function given by the fit routine to determine the time constant for this circuit.

*Time constant: _____*

Did the time constant increase, decrease or stay the same? How does this compare with the prediction you made prior to starting this activity?

**Question E3.2** Calculate the value of $RC$ with the new capacitor. Is the time constant you determined using the fit equal to the calculated value of $RC$?

**Prediction 3.2** How will the discharge of a capacitor change if the resistance it is connected to doubled? How will this affect the time constant?

**Step 5:** Increase the resistance to 20 ohms by adding a second 10 ohm resistor. (Use your knowledge from resistors in series and parallel to figure out the appropriate configuration for your two resistors i.e., in series or in parallel)

**Step 6:** Collect data and graph the values for current and voltage while the capacitor is being charged and discharged. Use the fit routine to determine the mathematical form of the discharging part of the graph, and write the formula with the values of the constants given by the fit routine below

**Question E3.3** Use the constant in the exponent of the function given by the fit routine to determine the time constant. Did the time constant increase, decrease or stay the same? How does the value obtained through the fit compare with your prediction?

*Time constant: _____*
This laboratory exercise closely follows the references below.